



Higher Moments of Net-proton Multiplicity Distributions at STAR

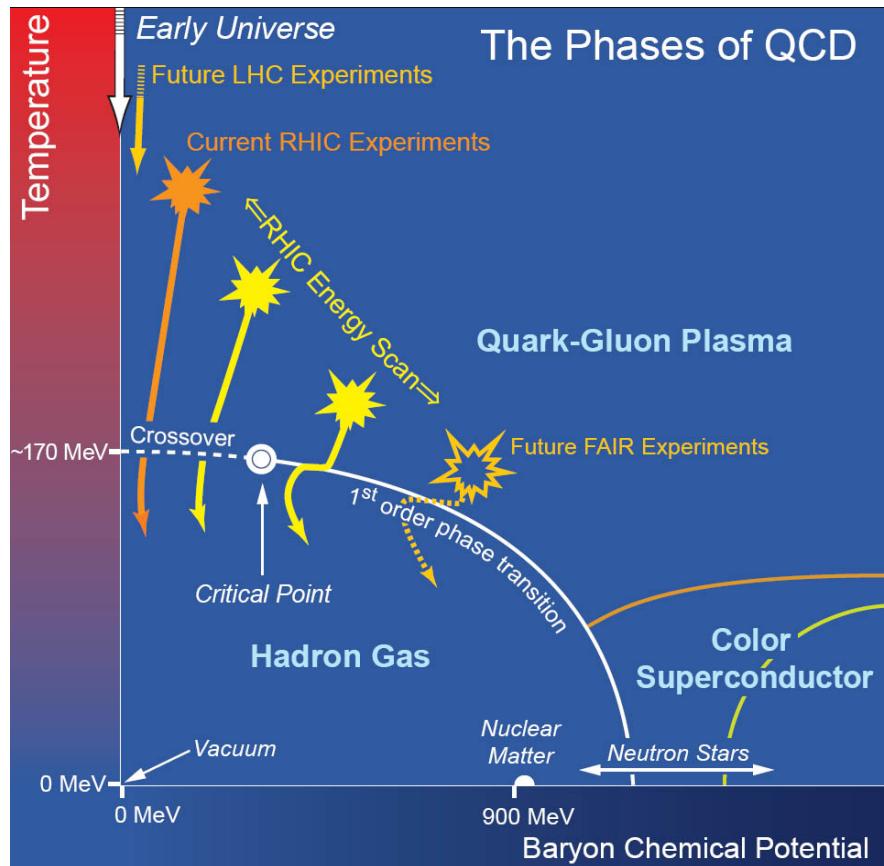


Xiaofeng Luo

Institute of Particle Physics, Central China Normal University

May/17/2012

QCD Phase Diagram



Lattice QCD:

- Crossover at $\mu_B = 0$, 1st order phase transition at large μ_B .

Y. Aoki et al., Nature 443, 675 (2006)
 S. Gupta, et al. Science 332, 1525 (2011).

- QCD Critical Point (CP): The end point of first order phase transition boundary.

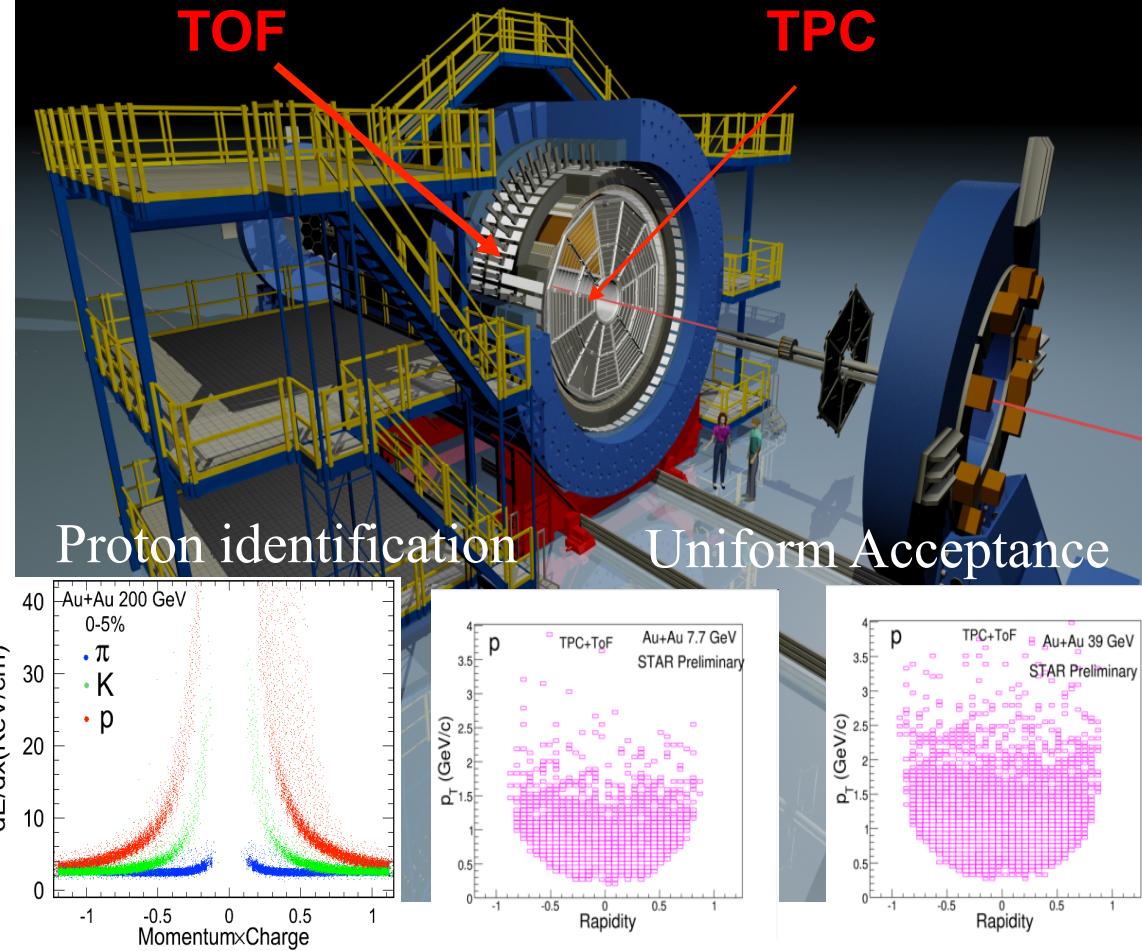
Z. Fodor, et al, JHEP04, 050 (2004) (hep-lat/0402006)
 M. A. Stephanov, Int. J. Mod. Phys. A 20, 4387 (2005) (hep-ph/0402115).

Main Goals of HIC:

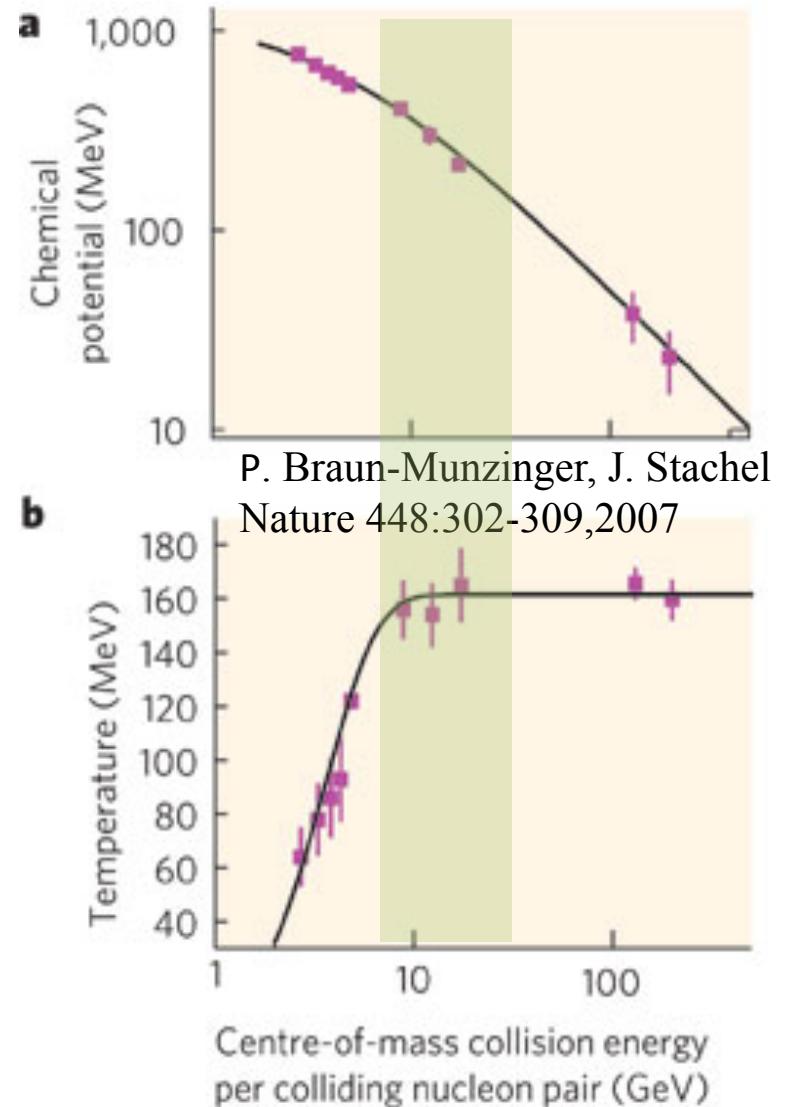
- Signals for phase transition/phase boundary.
- Search for Critical Point (CP).
- Bulk properties of QCD matter.

Experiment: Access the QCD Phase Diagram

Typical Experiment – STAR @ RHIC



Varying beam energy varies Temperature and Baryon Chemical Potential (RHIC Beam Energy Scan Program: 7.7, 11.5, 19.6, 27, 39, 62.4, 200 GeV)



Moments and Cumulants

In statistics, moments and cumulants are used to characterize the shape of the distributions.

$$\mu_n = \langle (N - \langle N \rangle)^n \rangle .$$

$$\mu_2 = \sigma^2, \mu_1 = 0$$

**Central
Moments**

Moments

$$\mu'_n = \langle N^n \rangle .$$

$$\mu'_1 = \langle N \rangle$$

$$C_n = \mu_n - \sum_{m=2}^{n-2} \binom{n-1}{m-1} C_m \mu_{n-m}$$

$$C_n = \mu'_n - \sum_{m=1}^{n-1} \binom{n-1}{m-1} C_m \mu'_{n-m}$$

Cumulants

$$C_1 = \langle N \rangle, C_2 = \mu_2 = \sigma^2$$

$$C_3 = \mu_3, C_4 = \mu_4 - 3\mu_2^2$$

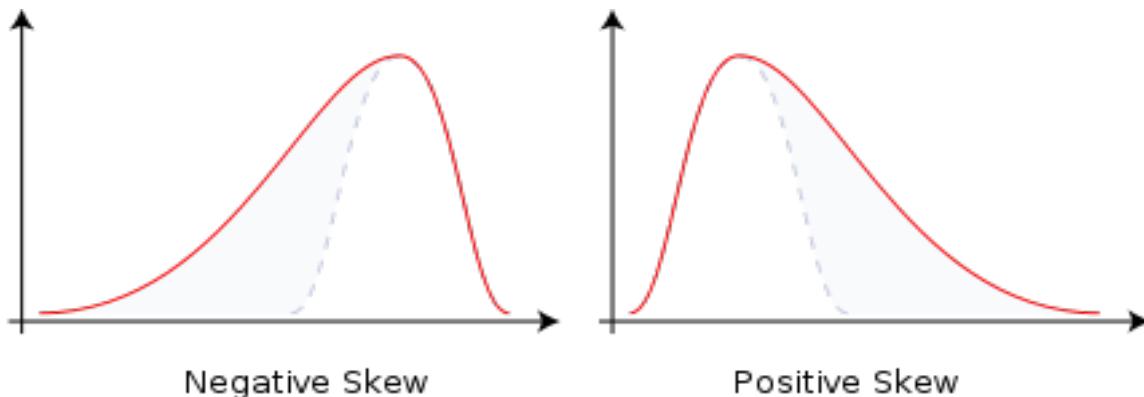
Skewness and Kurtosis

Normalized Central Moments : Skewness (3rd order) and Kurtosis (4th order).

Skewness:

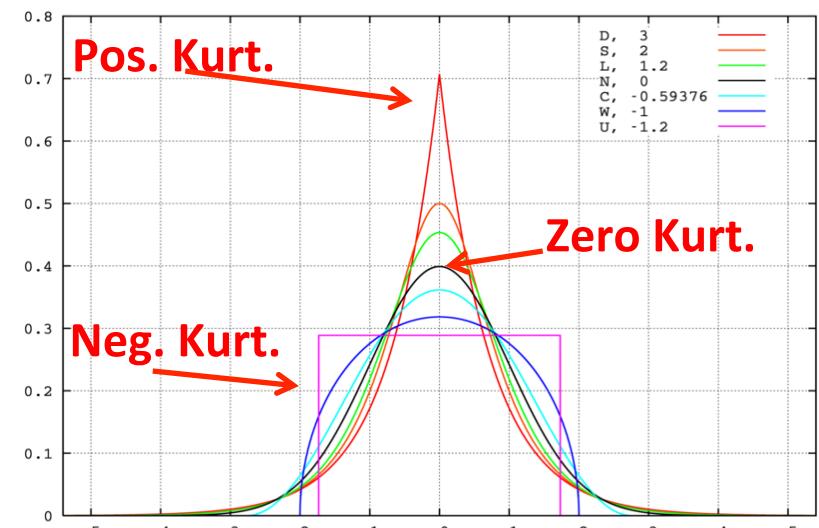
$$S = \frac{C_{3,N}}{(C_{2,N})^{3/2}} = \frac{\langle (N - \langle N \rangle)^3 \rangle}{\sigma^3}$$

N: Event by Event Multiplicity



Kurtosis:

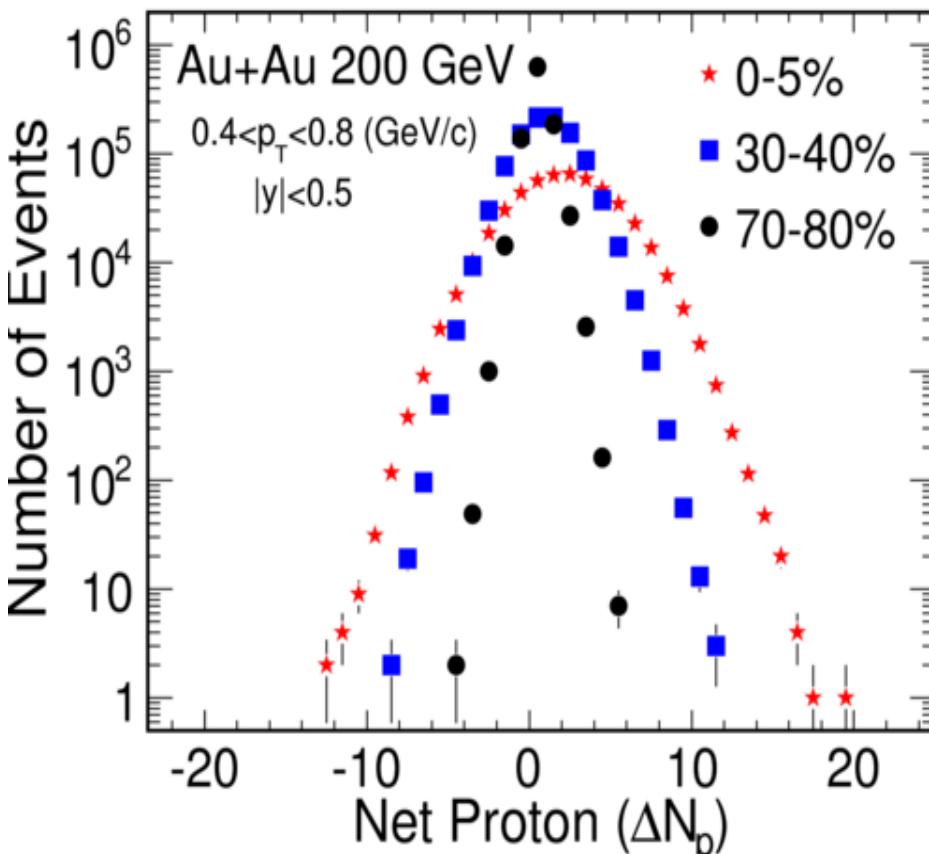
$$\kappa = \frac{C_{4,N}}{(C_{2,N})^2} = \frac{\langle (N - \langle N \rangle)^4 \rangle}{\sigma^4} - 3$$



- For Gaussian distribution, the skewness and kurtosis are equal to zero.
- Ideal probe of the non-Gaussian fluctuations.

Observable: Higher Moments of Net-proton Distributions.

Typical net-proton distributions



STAR: Physical Review Letters 105, 022302 (2010).

Describe: Shape and higher order fluctuations.

➤ Moments sensitive to correlation length (ξ):
Study **phase transition** and **Critical Point**.

$$\begin{aligned} \langle (\delta N)^2 \rangle &\sim \xi^2 & \langle (\delta N)^3 \rangle &\sim \xi^{4.5} \\ \langle (\delta N)^4 \rangle - 3 \langle (\delta N)^2 \rangle^2 &\sim \xi^7 \end{aligned}$$

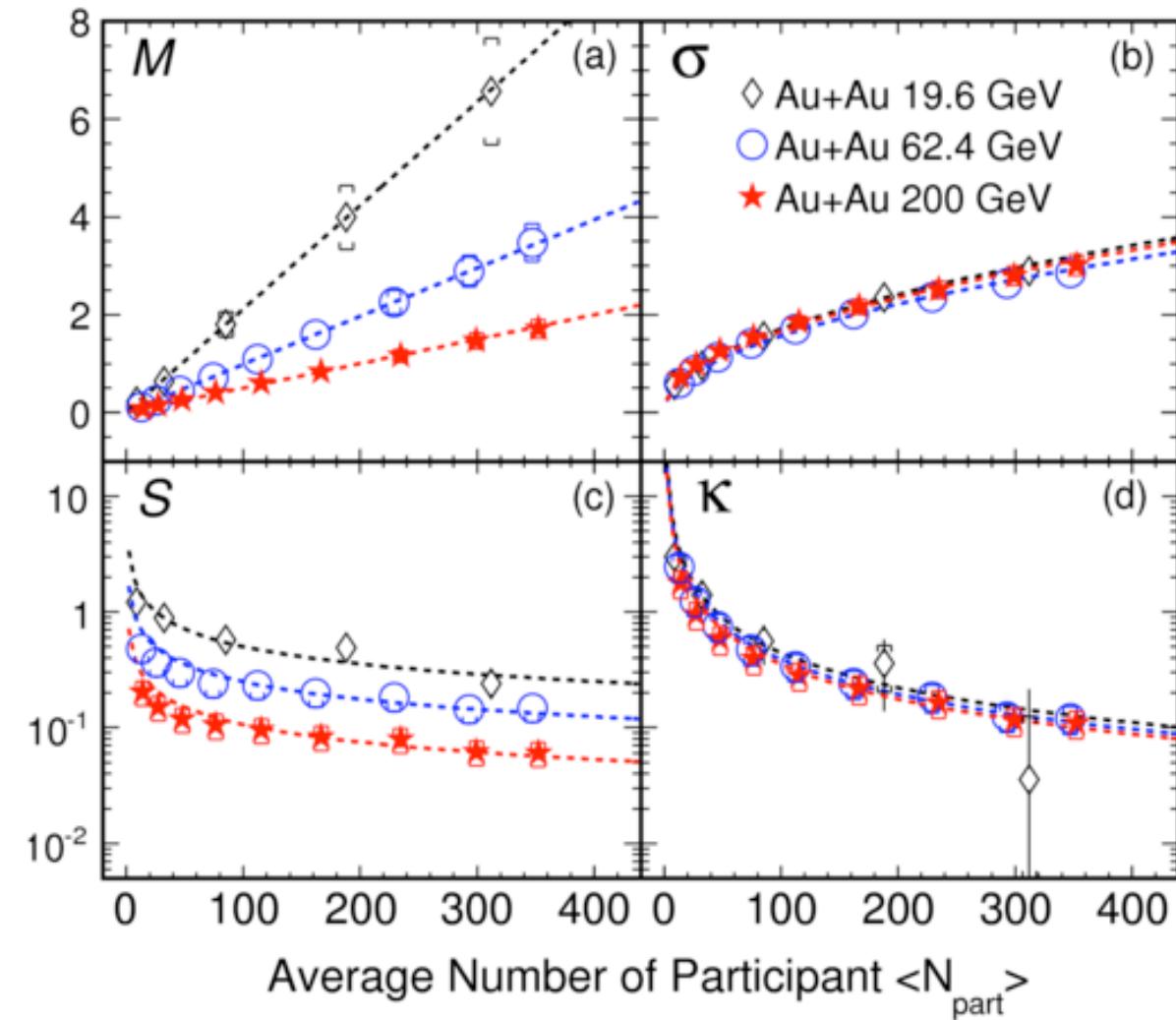
M. A. Stephanov, Phys. Rev. Lett. 102, 032301 (2009).
C. Athanasiou, et al., Phys. Rev. D 82, 074008 (2010)

➤ Moments products relates to baryon number Susceptibility ratio :
Study **Bulk properties** of QCD matter.

$\kappa \sigma^2 \sim \chi^{(4)}/\chi^{(3)}$ $S\sigma \sim \chi^{(3)}/\chi^{(2)}$ $\sigma^2/M = \chi^{(2)}/\chi^{(1)}$
Product of moments cancel volume effect.

F. Karsch et al, Phys. Lett. B 695, 136 (2011).
M. Cheng et al, Phys. Rev. D 79, 074505 (2009)

Moments of Net-proton Distribution



$$\sigma = \sqrt{\langle (N - \langle N \rangle)^2 \rangle}$$

$$s = \frac{\langle (N - \langle N \rangle)^3 \rangle}{\sigma^3}$$

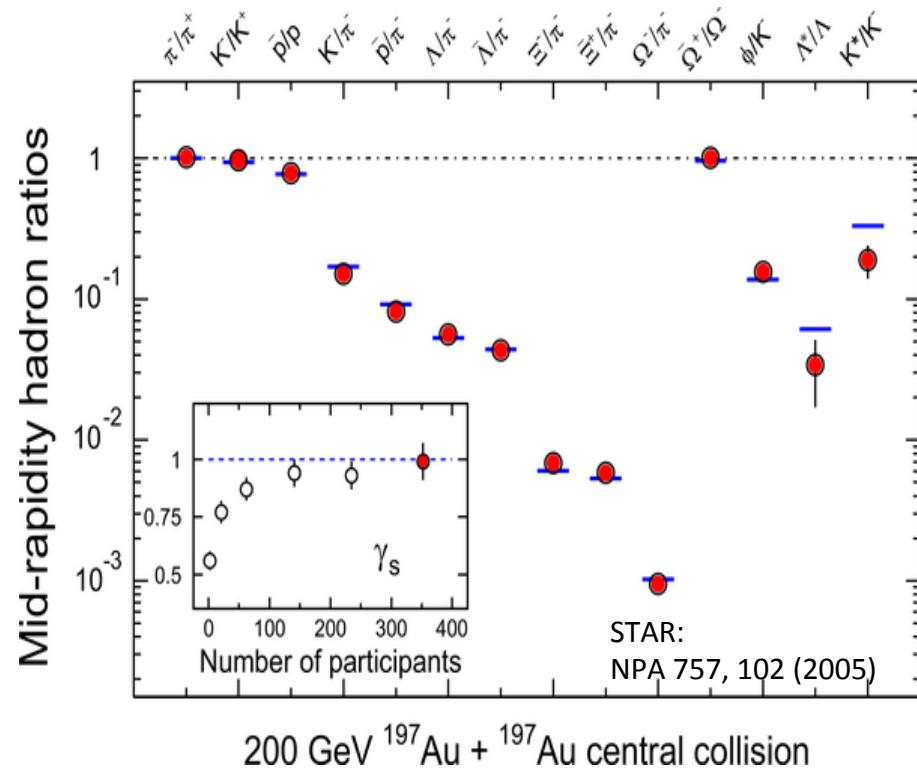
$$\kappa = \frac{\langle (N - \langle N \rangle)^4 \rangle}{\sigma^4} - 3$$

Dashed lines:
Central Limit Theorem (CLT)

Breaking down of the CLT
trends could indicate the QCD
critical point like physics effect.

Probing Chemical Freeze Out by HRG model

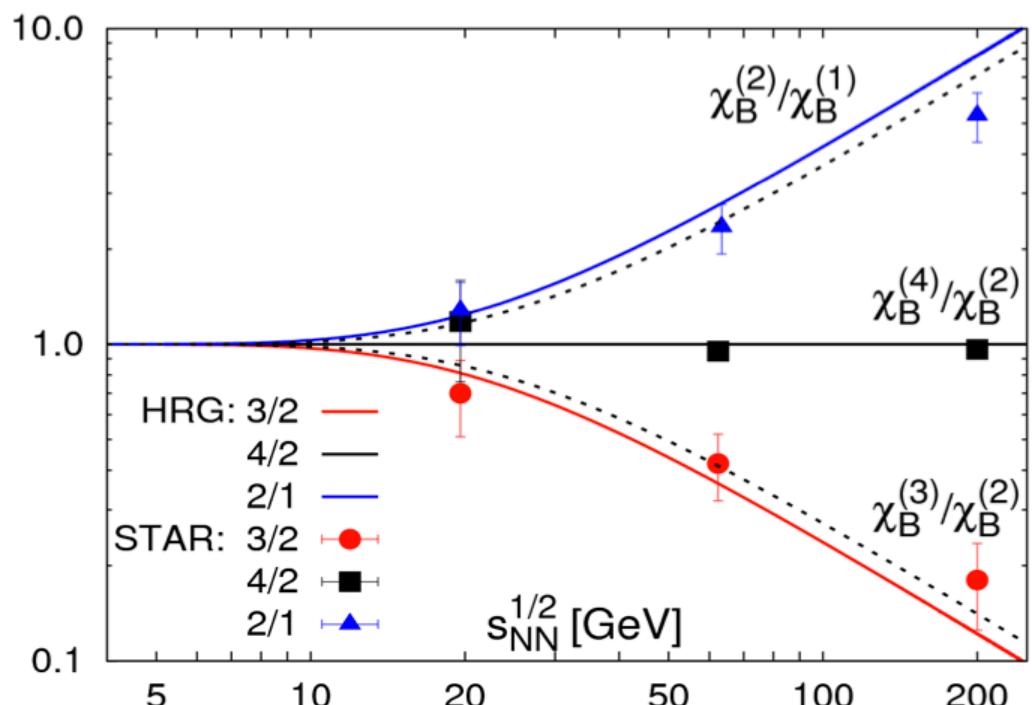
Success of Thermal model: First order



$$n_i = \frac{1}{2\pi^2} \int_0^\infty \frac{p^2 dp}{\exp[(E_i - \mu_i)/T] \pm 1}$$

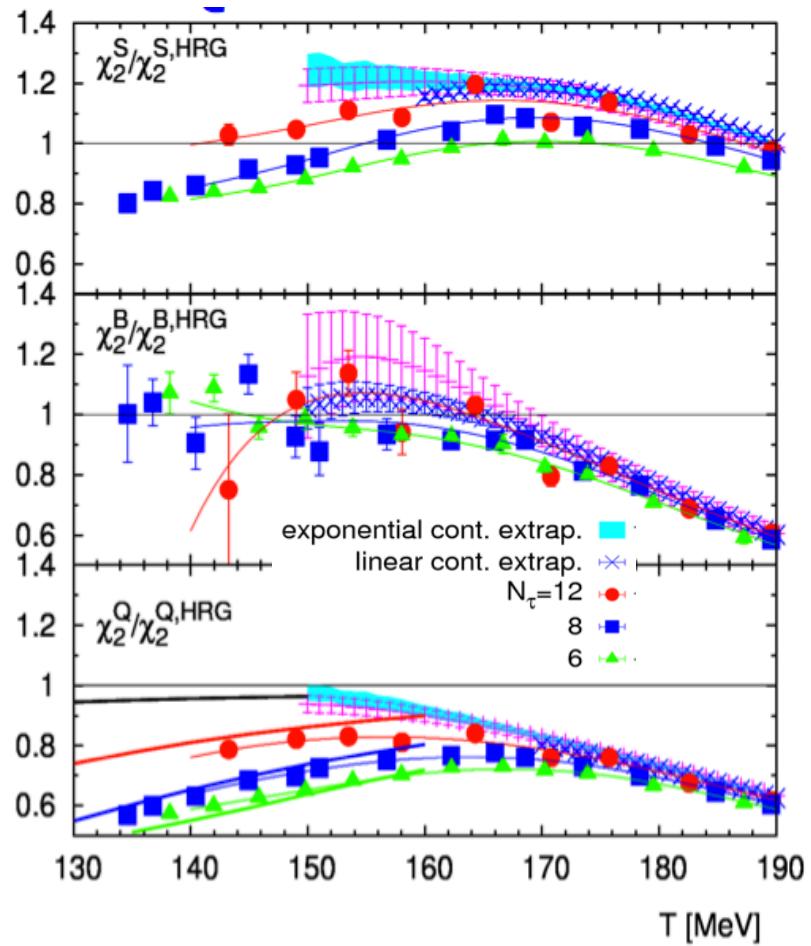
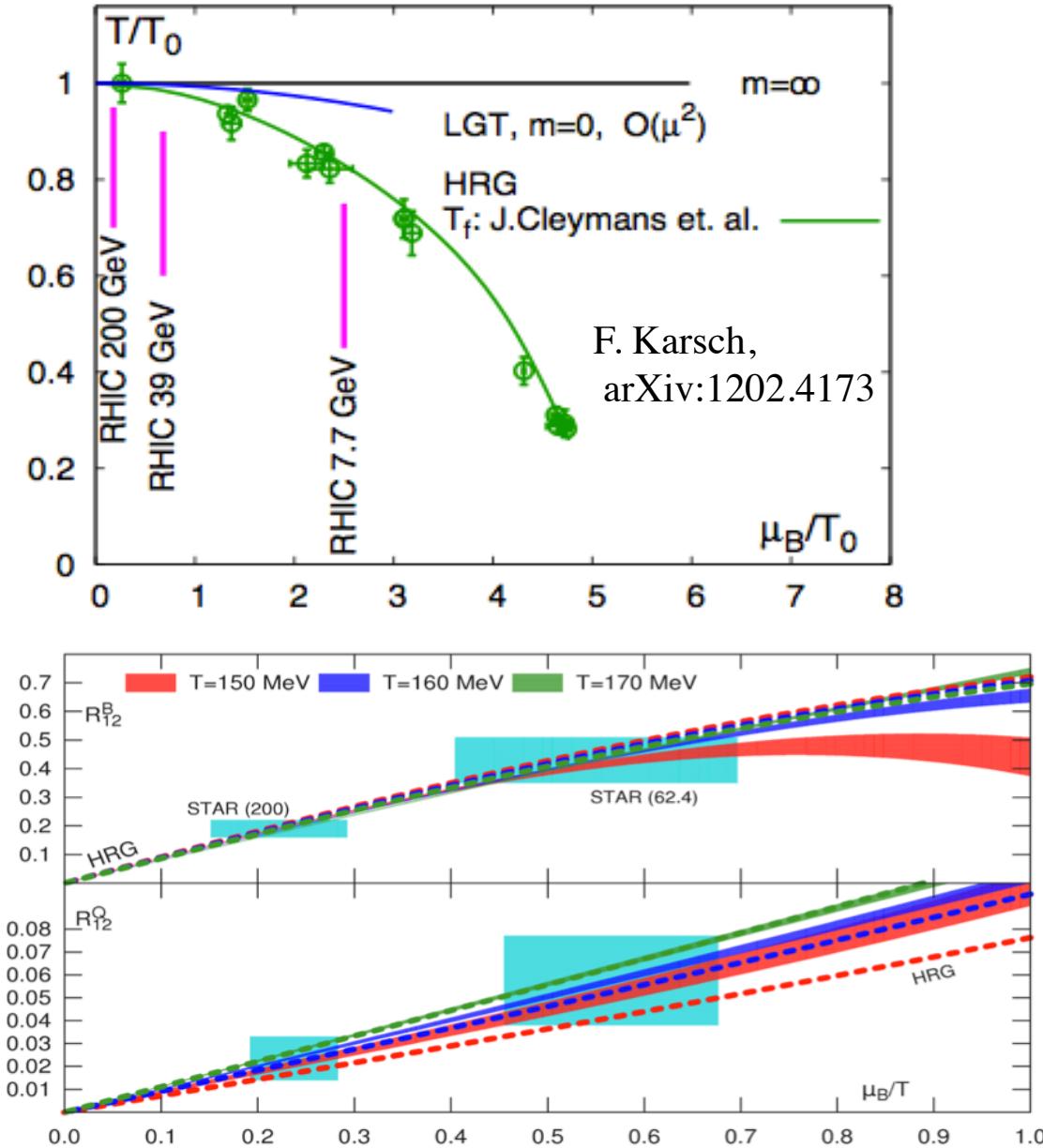
$$(S\sigma)_B = \chi_B^3 / \chi_B^2 = \tanh(\mu_B/T)$$

$$(\kappa\sigma^2)_B = \chi_B^4 / \chi_B^2 = 1$$



FK, K Redlich, arXiv:1007.2581

charge fluctuations at freeze-out agree well with HRG model predictions.



F. Karsch- LBNL 2012

Estimates of the chemical freeze-out parameters by Lattice QCD.

Love story : Chemical “Freak” out

Data

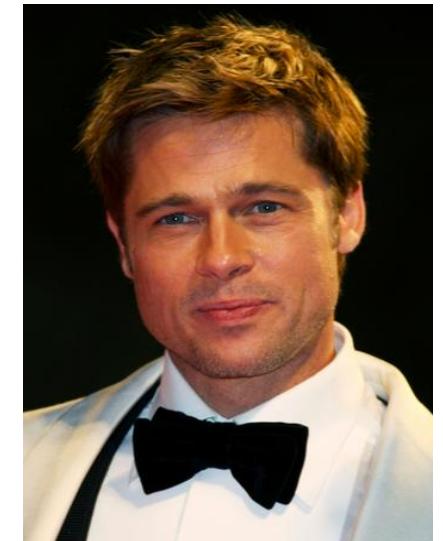


Who will send me a
Critical point ?

HRG



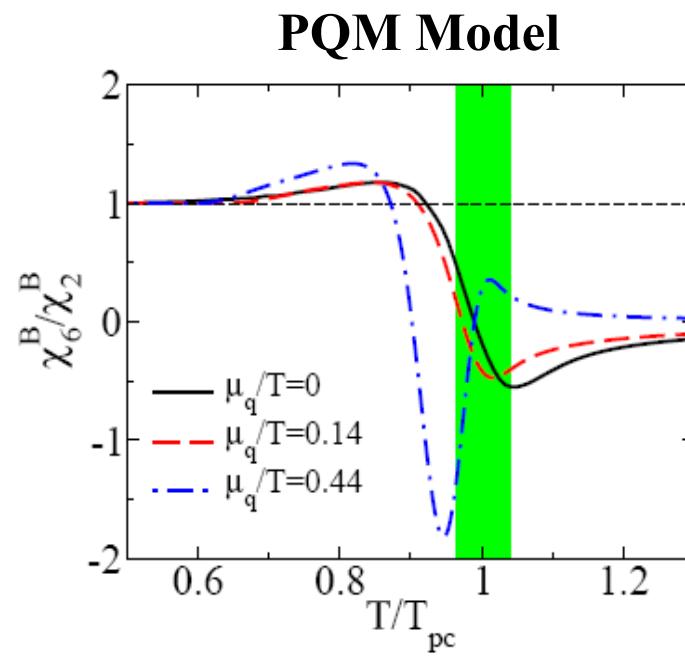
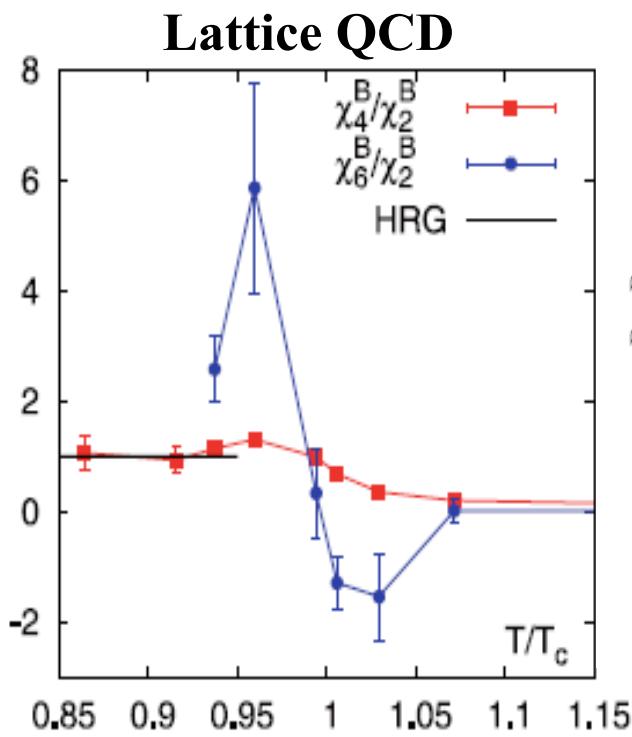
Lattice QCD



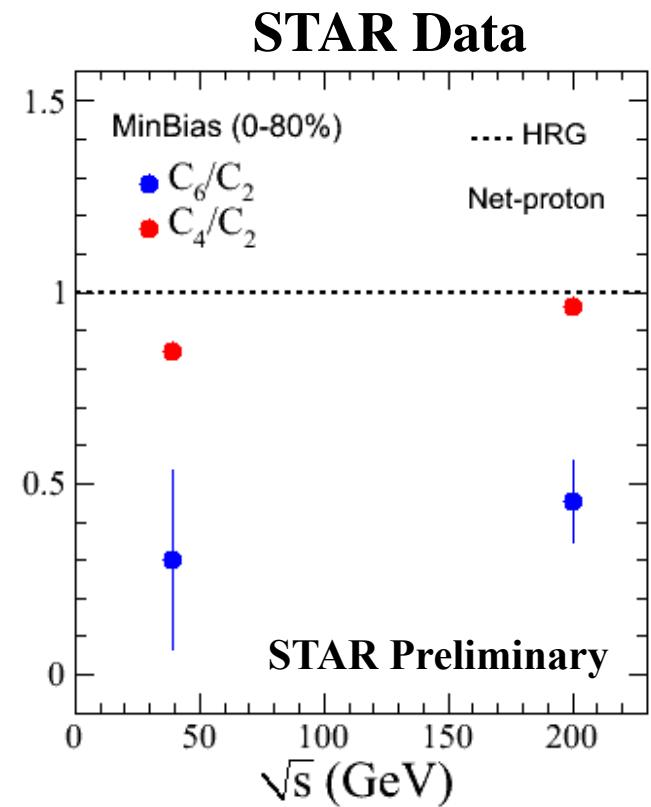
Fluctuations and Phase Transition

Deviation from HRG if freeze-out curve close to
Phase Boundary/Cross over line.

L. Chen, BNL workshop, CPOD 2011

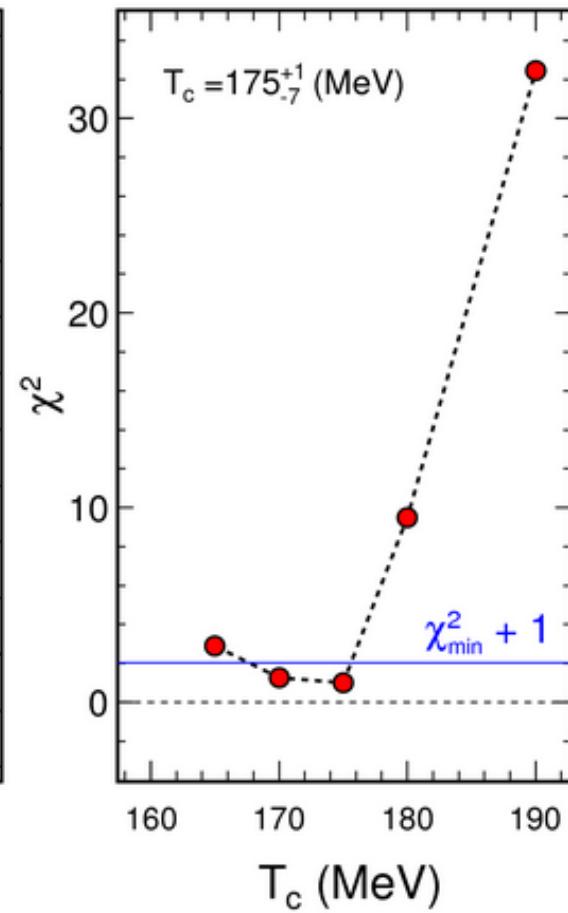
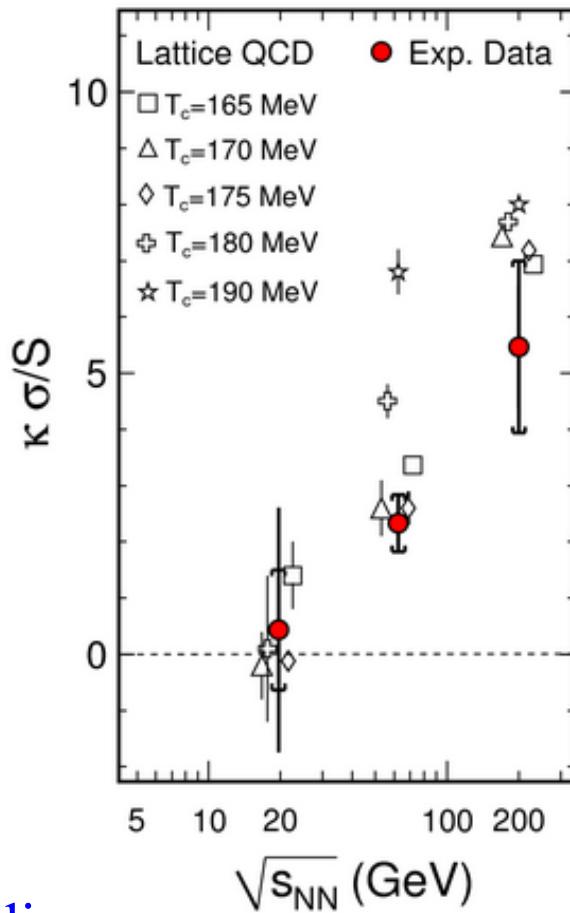
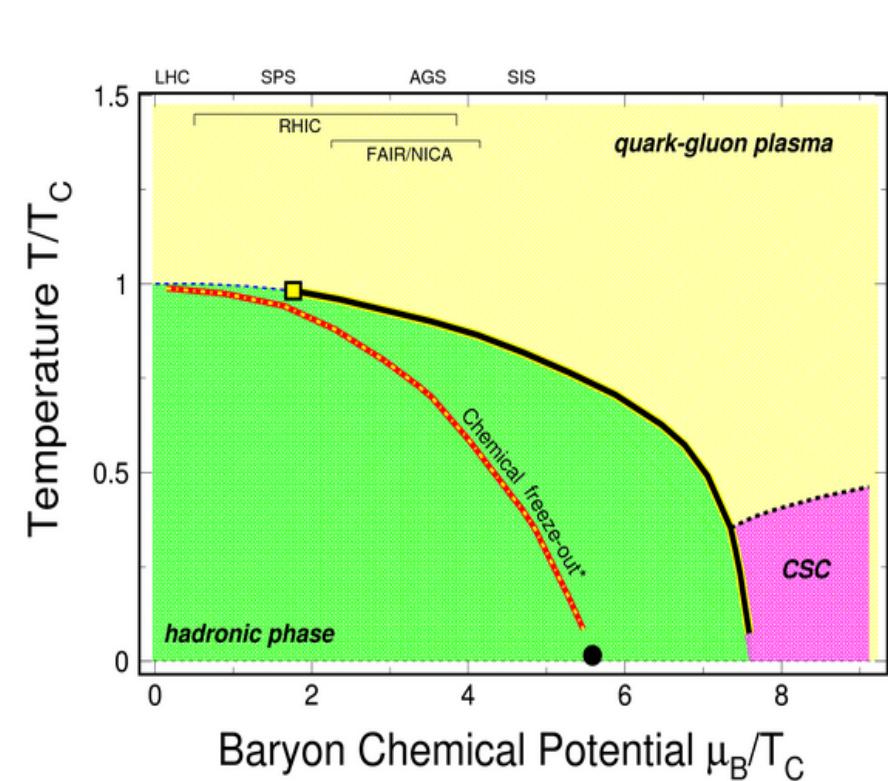


Redlich, LBNL 2012



Cheng *et al*, Phys.Rev. D79 (2009) 074505;
B. Friman *et al*., Eur. Phys. J. C 71 (2011) 1694

Fluctuations and Scale of the QCD Phase diagram

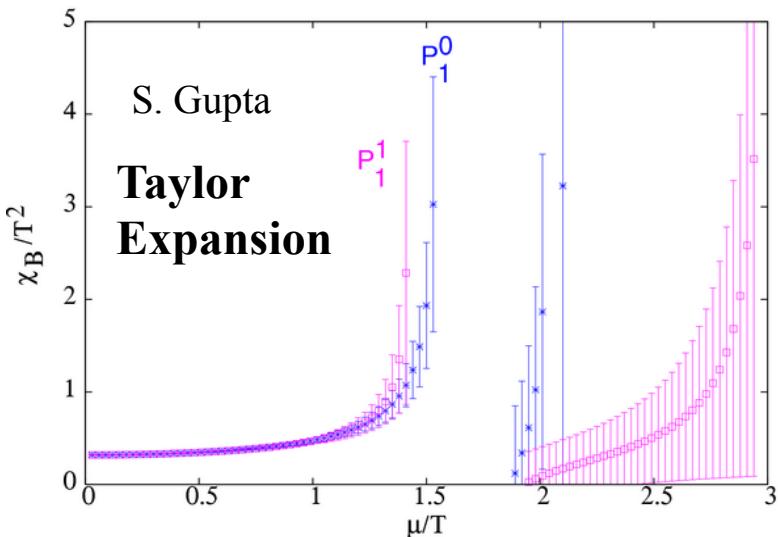
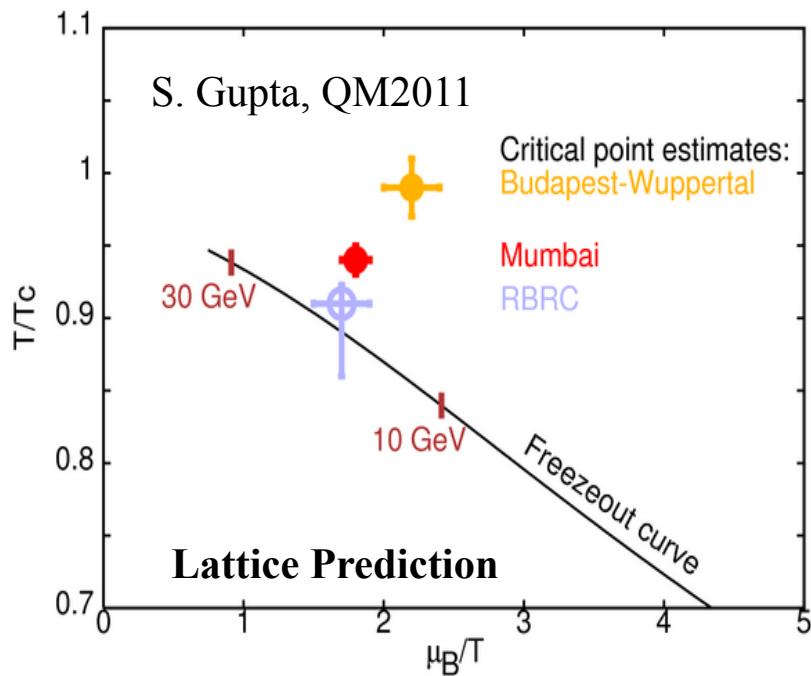


T_c sets the scale of the QCD phase diagram.

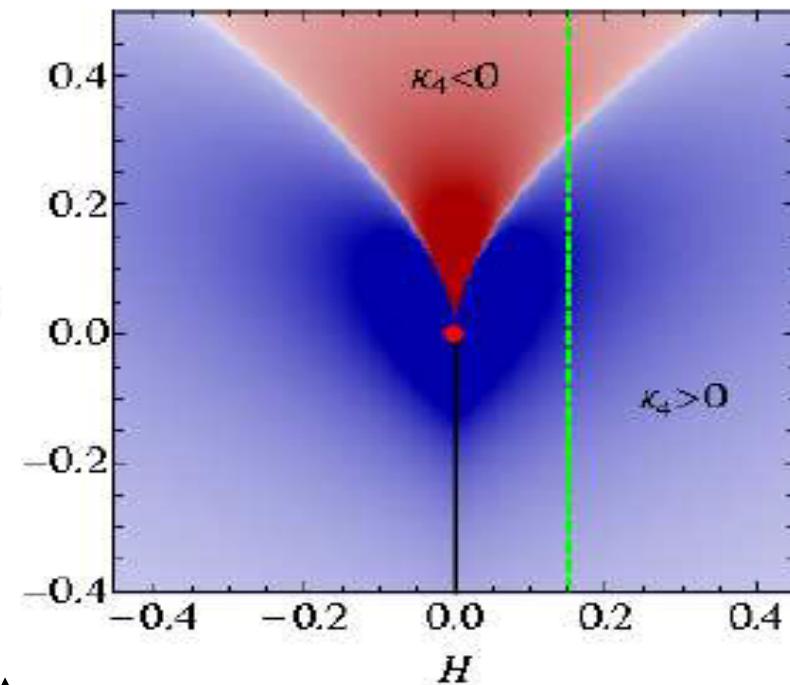
S. Gupta, X. Luo, B. Mohanty, H. G. Ritter, N. Xu,
Science 332 (2011) 1525.

$$T_c = 175^{+1}_{-7} \text{ MeV}$$

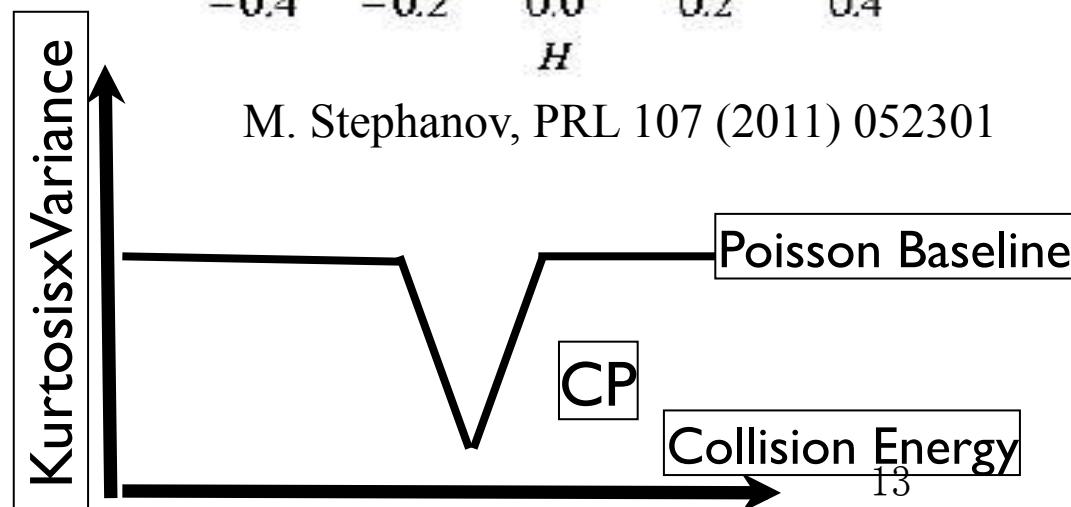
Fluctuations and Critical Point : Theoretical



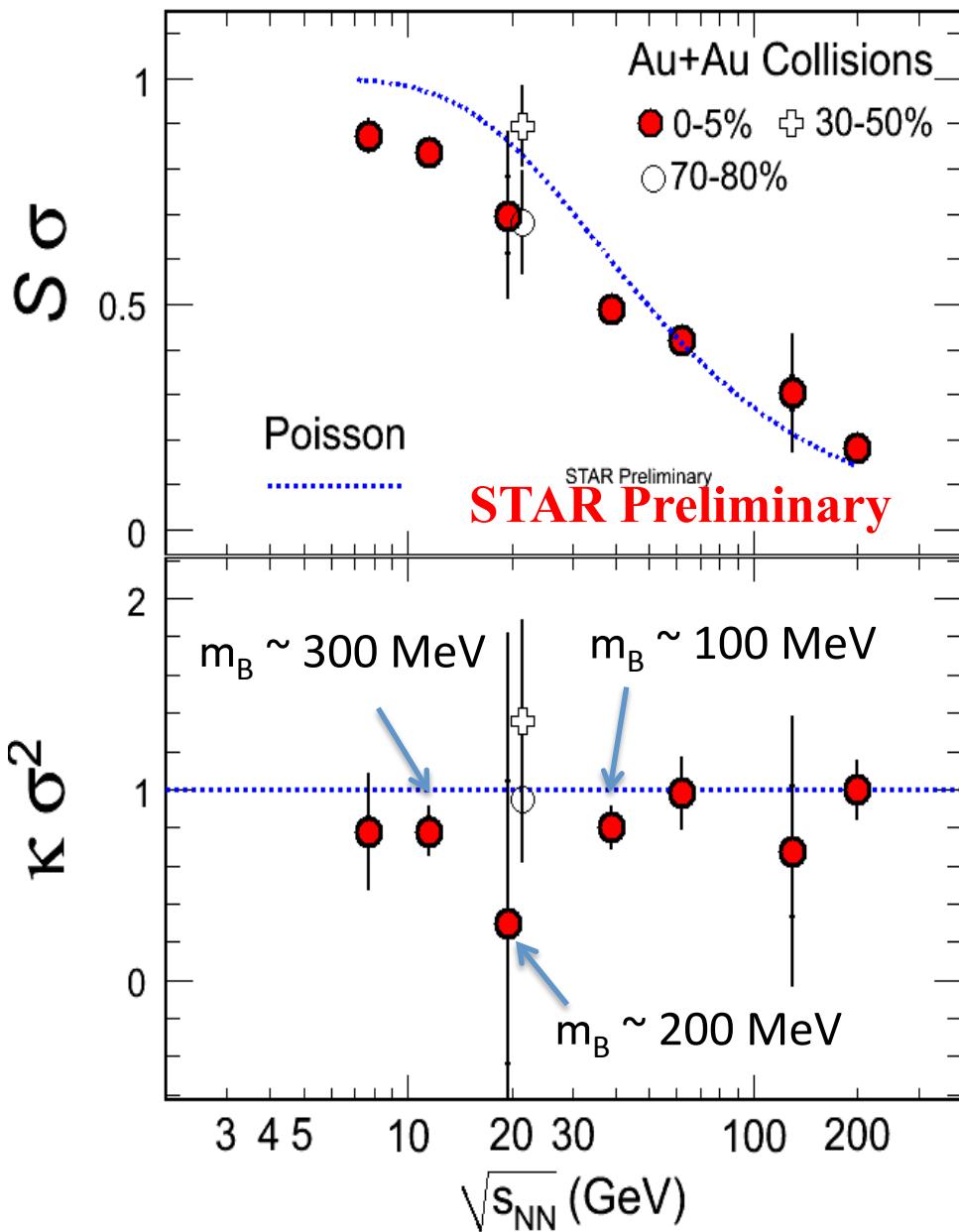
Kurtosis is negative when approach CP.



M. Stephanov, PRL 107 (2011) 052301



Preliminary results from STAR BES Data



STAR: QM2011 + X. F. Luo CPOD 2011

Experimental results:

- Effect of auto correlations (small at 200 GeV could be large at 7.7 GeV).
- Study the effect of conservation law.
- More accurate error estimates.

X. Luo, JPG 39, 025008 (2012)
[arXiv: 1109.0593]

Will have more precise results at 19.6 GeV with larger statistics and new energy point at 27 GeV.

Finalized results will come out soon and be presented at QM2012.

Summary and Outlook

Summary:

- Higher moments are sensitive to the correlation length.
Search for the signal of phase transition and critical point.
- Higher moments are related to the susceptibility.
 1. **Study the bulk properties and thermodynamics of QCD matters. Probe the chemical freeze out conditions.**
 2. **Test of the non-perturbative QCD.**

Outlook:

- Will soon complete the higher moments of net-protons measurements for RHIC BES program.
- Comparison with HRG, Lattice QCD results etc...



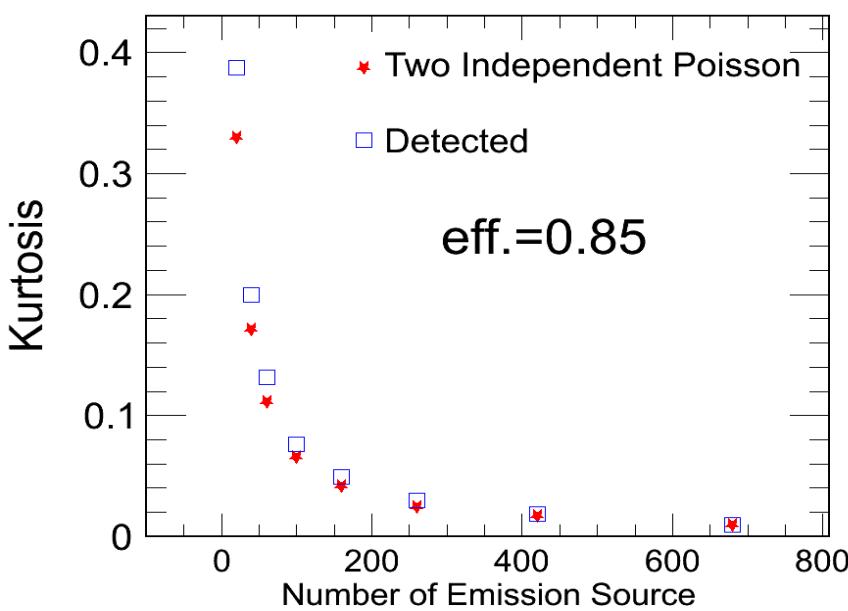
Backup slides

Detector Efficiency Effect

➤ **Binomial Process of Detected Particles:** With the total produced multiplicity N and the detector efficiency ε .

$$B(n; N, \varepsilon) = \frac{N!}{n!(N-n)!} \varepsilon^n (1-\varepsilon)^{N-n} \quad \rightarrow \quad T(k) = \sum_N B(k; N, \varepsilon) P(N)$$

➤ **Monto Carlo:** Input two Independent Poisson Distribution: $N=N_1-N_2$

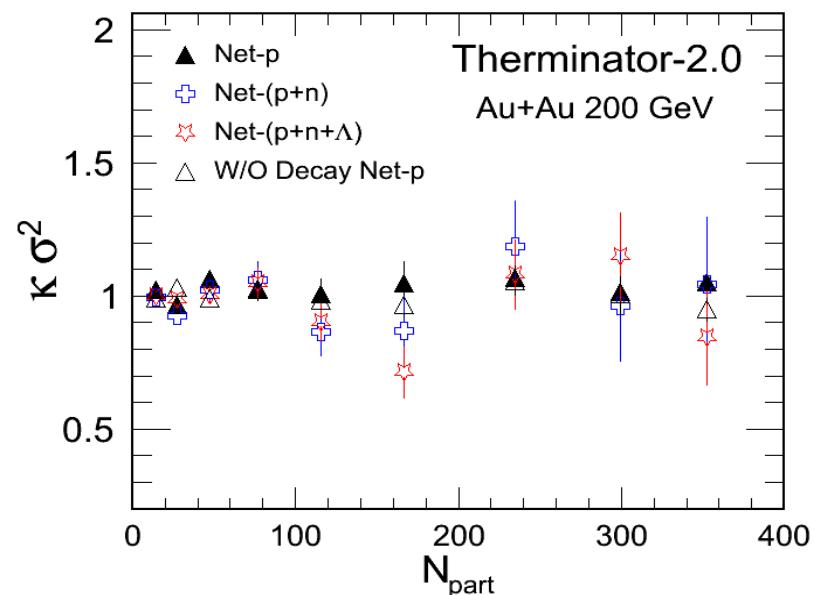
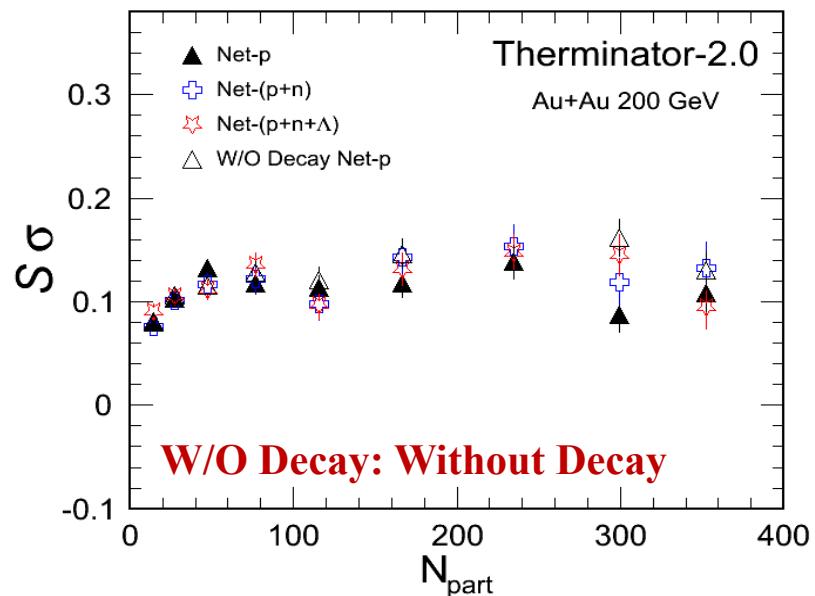
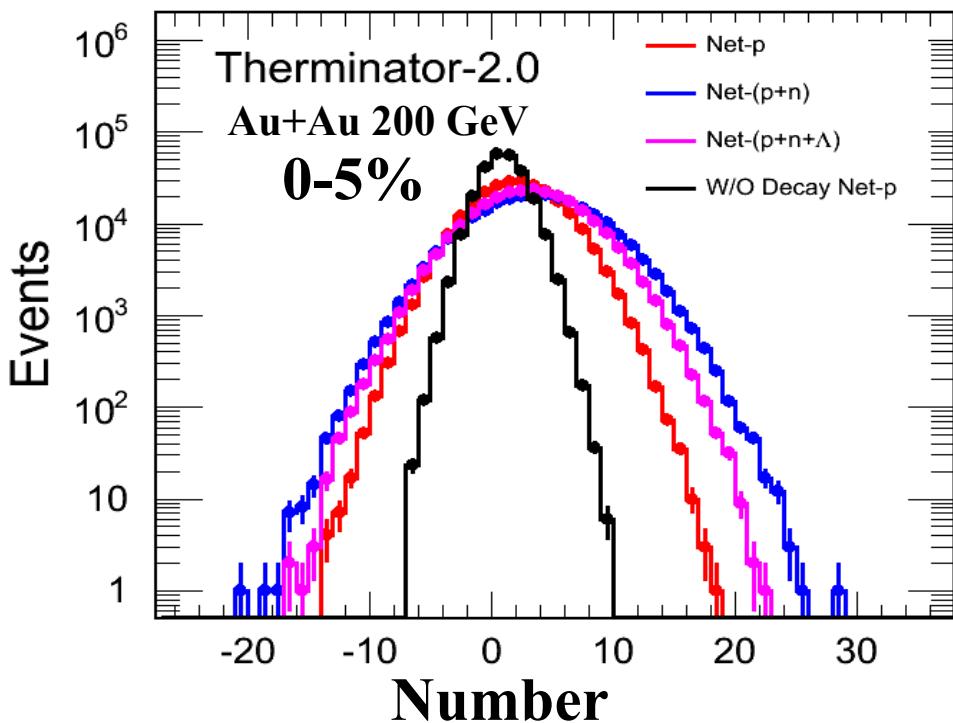


$$\begin{aligned} \sigma_k^2 &= \varepsilon \sigma_N^2 \\ S_k &= \frac{S_N}{\sqrt{\varepsilon}} \\ K_k &= \frac{K_N}{\varepsilon} \end{aligned} \quad \rightarrow \quad \begin{aligned} S_k \sigma_k &= \frac{S_N}{\sqrt{\varepsilon}} * \sqrt{\varepsilon} \sigma_N = S_N \sigma_N \\ K_k \sigma_k^2 &= \frac{K_N}{\varepsilon} * \varepsilon \sigma_N^2 = K_N \sigma_N^2 \end{aligned}$$

➤ For this case, the efficiency effects are simple and will be cancelled out for moment products.

Resonance Decay and Neutron Effect

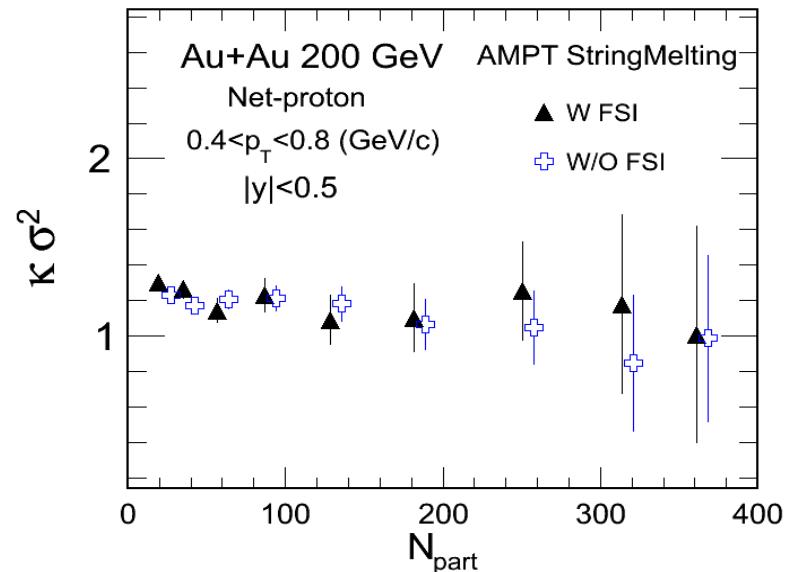
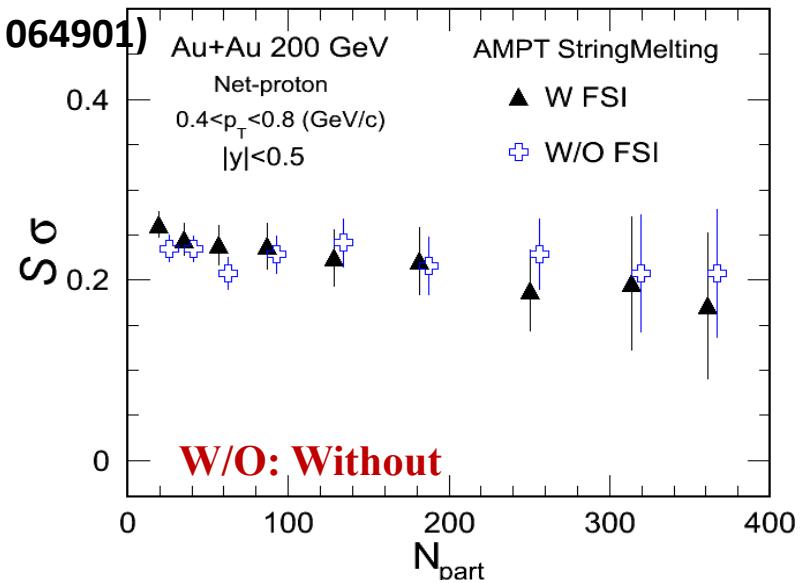
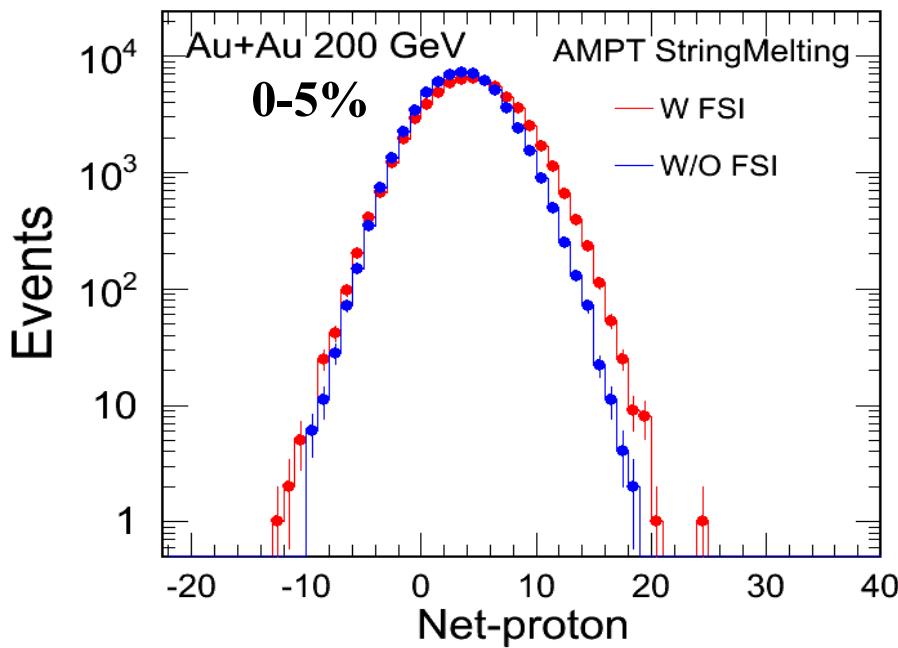
Model: Therminator-2.0 (arXiv:1102.0273)



- Effect of **resonance decay** on $S\sigma$ and $\kappa\sigma^2$ is small. (based on the right two plots).
- Effect of inclusion of neutrons is small: Indicates: **Net-proton fluctuation can reflect the net-baryon fluctuation.**
- Error estimation: X. Luo, arXiv:1109.0593

Final State Interaction (FSI) Effect

Model: AMPT StringMelting (Phys. Rev. C 72, 064901)



- Process Final State Interaction (FSI) between hadrons or not can be controlled by “ART” program in the AMPT model.
- Effects of Final State Interaction (FSI) on $S\sigma$ and $\kappa\sigma^2$ are small.

(based on the results in the right two plots).